Substratum Approach to a Unified Theory of Elementary Particles

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If special relativity is a dynamic symmetry caused by true physical deformations of bodies in absolute motion through a substratum or ether, the question if all interactions and elementary particles are excitations of this ether must be raised. The ether being the cause of all the observed relativistic effects should then obey an exactly nonrelativistic law of motion, and which permits it to consist of positive and negative masses. The fundamental constants of nature, which according to Planck are 1) Newton's constant (G), 2) the velocity of light (c) and 3) Planck's constant ($\hbar$), suggest that the ether is made up of densely packed positive and negative Planck masses (Planckions), each with a diameter equaling the Planck length. Symmetry demands that the number of positive and negative Planck masses should be equal, making the cosmological constant equal to zero. Because the Planckions are nonrelativistic spin-zero bosons, the ether would therefore consist of two superfluids, one for the positive mass Planckions, and the other one for the negative mass Planckions. By spontaneous symmetry breaking this superfluid ether can in its ground state form a lattice of small vortex rings, with the vortex core radius equaling the Planck length. Force fields of massless vector gauge bosons can be interpreted as quantized transverse vortex waves propagating through this lattice. Because the smallest wave length would be about equal the ring radius of the circular vortices, the ring radius would assume the role of a unification scale. The ring radius is estimated to be about 10^3 times the Planck length, in fairly good agreement with the empirical evidence for the value of the grand unification scale of the standard model.

Charge is explained by the zero point fluctuations of the Planckions attached to the vortex rings, which thereby become the source of virtual phonons. Charge quantization is explained as the result of circulation quantization. Spinors result from bound states of the positive and negative masses of the substratum, and special relativity as a dynamic symmetry would be valid for all those objects. Quantum electrodynamics is derived as a low energy approximation

If spinors are made up from the positive and negative masses of the vortex ring resonance energy, whereby the spinors would assume the character of excitons, the spinor mass can be computed in terms of the Planck mass. Vice versa, with the lowest quark mass m given, a value for the gravitational constant in terms of m, $\hbar$, and c can be obtained. The existence of different particle families can be understood by internal excitations of the spinors, and parity violation may find its explanation in a small nonzero vorticity of the ether. Bacause of its simple fundamental symmetry the theory is unique, it is always finite and has no anomalies.

In the proposed theory all fields and interactions are explained in a completely mechanistic way by the Planck masses and their contact interactions. With special relativity as a derived dynamic symmetry and space remaining euclidean, the proposed approach can be seen as an alternative to Einstein's program to explain all fields and their interactions by symmetries and singularities of a noneuclidean spacetime manifold.

In Part I, the fundamental equation for the substratum, which has the form of a nonrelativistic nonlinear Heisenberg equation, is formulated. It is shown how it leads to a Maxwell-type set of equations for the gauge bosons. In Part II, Dirac-type spinors and quantum electrodynamics are derived. These results are then applied to obtain the lowest quark mass in terms of the Planck mass.

Part I

1. Introduction

Almost 100 years ago, and prior to the discovery of quantum mechanics, attempts were made to explain all force fields in a purely mechanistic way. The Newtonian action at a distance doctrine was rejected, and replaced by the hypothesis that all forces are transmitted through space filled with an ether, which everybody believed to exist after it was discovered that light is a wave transmitted through space. The goal of this program was to eliminate the force fields by reducing them to the "thrust and pressure" of the engineer, with the forces transmitted through collisions in between adjacent ether atoms. After Hermann von Helmholtz [1] had shown that vortex rings would be everlasting in a frictionless fluid, Sir William Thomson (Lord Kelvin) made the hypothesis that the ether is an incompressible frictionless fluid. He then proved [2], that if this ether forms a lattice of vortex rings it would

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be capable to transmit the kind of waves predicted by Maxwell's equations, provided the wavelength is large compared with the ring radius of the vortices. These very promising attempts were brought to an abrupt end by Einstein's rejection of the ether and its replacement by his well-known postulates, which were motivated to explain the negative outcome of the Michelson-Morley and other experiments. Years later though, it was recognized that quantum mechanics after all leads to a kind of ether, the zero point energy of the vacuum. Its existence is empirically confirmed, for example, through the well-known phenomenon of spontaneous emission. To be compatible with special relativity, this zero point energy must have a ω^3 frequency spectrum, and therefore is infinite. An infinite energy, of course, because it would lead to infinite gravitational forces, is in gross disagreement with everyday experience. Supersymmetric theories, including superstring theories, with the zero point energies from the Fermi and Bose sector cancelling out against each other, avoid this infinity, but the problem there is that the supersymmetric particles have never been found, at least up to presently known particle energies. This means that the zero point energy left uncompensated, in case these particles are eventually found at some high energy, would still be huge, producing large gravitational fields, which are not observed.

Already prior to the discovery of the laws of quantum mechanics firmly establishing the existence of a zero point energy ether, it was Fitzgerald, followed by Lorentz and Poincaré who insisted that the Lorentz transformations, and hence all of special relativity, could be explained as well by assuming that bodies in absolute motion with a velocity v through a substratum are contracted by the factor $\sqrt{1-v^2/c^2}$, and that all clocks moved along would go slower by the same factor. The anisotropy in the propagation of light in a comoving reference system would there be unobservable because in reality one can measure only the to and fro velocity, a fact which even finds its expression in the Lorentz transformations, containing only the scalar c^2, rather than the vector $\boldsymbol{c}$. The contraction hypotheses, which at first seemed to be very artificial, was later made quite plausible by Lorentz who derived it for bodies held together by electromagnetic forces, assuming that Maxwell's equations are valid in a system at rest with the ether. The slowing down of clocks would there then follow from the contraction effect, because if held together by electromagnetic forces, all clocks would behave like light clocks*. From this perspective the negative outcome of the Michelson-Morley experiment is almost trivial, because if the arms of the interferometer are held together by electromagnetic forces they must suffer the same deformation as the light paths, thereby cancelling each other out. In this alternative dynamic interpretation of special relativity, the Lorentz invariance of Maxwell's equation is explained as an illusion caused by the measurement with contracted rods and slower going clocks. The contraction, however, would now have to take a finite time, and within shorter times Lorentz invariance would be violated. For elementary particles this time would have to be very short to explain why for them the laws of special relativity are so well satisfied. For macroscopic objects this must not necessarily be true, and a small siderial tide observed on the rotating earth with a superconducting gravimeter could possibly be interpreted as such a non-adiabatic relativity-violating effect [3].

If the ether is to be identified with the quantum mechanical zero point energy of the vacuum it must be formulated by quantum mechanical principles. A return to the classical mechanical ether models of the last century is out of question.

The idea that Lorentz invariance is a dynamic symmetry caused by the interaction of all bodies with an ether leads to the plausible hypothesis that the ether as the cause of all the relativistic effects should obey a nonrelativistic law of motion exactly. The assumption of a nonrelativistic law of motion makes it possible to introduce negative masses in addition to positive masses, because in nonrelativistic quantum mechanics the particle number operator commutes with the Hamilton operator.

The fundamental constants of nature G, c, and $\hbar = h/2\pi$ then suggest that the ether or substratum is composed of Planck mass particles (Planckions) with a mass m_p equal to

$$m_p = \pm\sqrt{\hbar c/G} \simeq \pm 2.2 \times 10^{-5}\ \mathrm{g} \tag{1.1}$$

and with a size r_p equal to

$$r_p = \sqrt{\hbar G/c^3} \simeq 1.6 \times 10^{-33}\ \mathrm{cm}. \tag{1.2}$$

* A light clock consists of a rod with two mirrors attached to its ends in between which a light signal is reflected back and forth.

The expressions for m_p and r_p follow from the two fundamental quantum mechanical relations:

$$m_p r_p c = \pm h, \qquad (1.3)$$

$$G m_p^2 = h c, \qquad (1.4)$$

where we have permitted both signs for the mass m_p. For reasons of symmetry we request that the substratum consists of an equal number of positive and negative Planck masses in a densely packed assembly. The assumption of an equal number of positive and negative mass Planckions retains the zero point energy fluctuations of the vacuum, but makes the average to vanish. An exactly equal number of positive and negative Planck masses makes the cosmological constant exactly equal to zero, in very good agreement with the empirical evidence establishing a very low upper bound for this constant.

2. The Fundamental Equation for the Substratum

In keeping with the doctrine that all forces are transmitted through "thrust and pressure" we postulate a delta-function potential in between the Planckions:

$$V(\mathbf{r}) = \pm f^2 r_p^2 \delta(\mathbf{r}), \qquad (2.1)$$

where r is the distance between two Planckions, and f a coupling constant. Between Planckions possessing the same sign the potential is repulsive, and between Planckions of opposite sign it is attractive.

The quantized version of the many body problem for the Planckions interacting with each other by a delta function potential reduces to the Heisenberg equations for the field operators ψ_+ (ψ_+ for positive mass and ψ_- for the negative mass Planckions):

$$i\hbar \frac{\partial \psi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_p} \nabla^2 \psi_\pm + f^2 r_p^2 (\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-) \psi_\pm \qquad (2.2)$$

with the commutation relations

$$[\psi_\pm(\mathbf{r})\, \psi_\pm^\dagger(\mathbf{r}')] = \delta(\mathbf{r}-\mathbf{r}'),$$
$$[\psi_\pm(\mathbf{r})\, \psi_\pm(\mathbf{r}')] = [\psi_\pm^\dagger(\mathbf{r})\, \psi_\pm^\dagger(\mathbf{r}')] = 0. \qquad (2.3)$$

We claim that (2.2) and (2.3) have all the ingredients to establish: 1) Vector gauge bosons, 2) charge and charge quantization, 3) special relativity as a dynamic symmetry, 4) gauge invariance, 5) Dirac-type spinors, 6) the mass of a typical elementary particle expressed in terms of the Planck mass, 7) quantum electrodynamics.

The symmetry underlying (2.2) is very simple. It is the Galilei group and the $U(2) = U(1) \times SU(2)$ group of two fundamental objects, the two Planckions of opposite sign. It happens that the $SU(2)$ group is isomorphic with the rotation group in three dimensions, which is part of the full Galilei group. The ψ^3-term in (2.2) occurs also in the Higgs field of the standard model, where it is responsible for generating mass.

We would like to remark that (2.2) is similar to Heisenberg's nonlinear spinor equation [4], but for three important reasons it is different. First, (2.2) is nonrelativistic, second, it describes a scalar, and third, it has two possible mass signs. Heisenberg's attempt to formulate a unified theory of elementary particles with his nonlinear spinor equation failed because of unavoidable divergences. No such divergences can occur with the nonrelativistic wave equation (2.2).

The proposed theory fulfills v. Weizsäcker's [5] two postulates for a fundamental theory: 1) The finiteness postulate, fulfilled by a smallest length, the Planck length; and 2) the postulate of the ultimate alternatives, fulfilled by the two signs of the Planck masses. However, in contrast to v. Weizsäcker's unification postulates, neither Lorentz invariance nor the gauge principle are needed here as additional postulates. As we will show, they can both be derived as dynamic symmetries instead.

We would like to add a remark regarding the origin of the inertial forces, which in a nonrelativistic theory require Newton's hypothesis of an absolute space. For the establishment of (2.2) no such hypothesis is needed because the negative mass ether acts like an absolute space for the positive mass ether and vice versa. We, therefore, can say that the inertial forces are a direct consequence of the nonlinearity of (2.2), coupling the positive with the negative mass ether.

3. The Hartree Approximation

Equation (2.2) can be interpreted as an operator equation for two coupled Bose-fluids, one composed of the positive and the other one of the negative mass Planckions. In the ground state $|0\rangle$ a macroscopic number $n_0 = 1/r_p^3$ of Planckions per unit volume and of both signs has zero momentum, resulting in two superfluid condensates, one for the positive and one for the negative mass Planckions. According to Bogolyubov's prescription, the existence of such a highly degenerate ground state permits one to neglect fluctuations in the condensate, replacing the creation operator $a^\dagger$ by the c-number $\sqrt{n_0}$. With this approximation one obtains a new, simplified set of equations of motion in which individual Planckions are driven by the condensate. We then can replace the wave function of the condensate by the expectation value of (2.2) with

$$\varphi_\pm \equiv \langle \psi_\pm \rangle, \qquad \varphi_\pm^* \equiv \langle \psi_\pm^\dagger \rangle. \tag{3.1}$$

To express the nonlinear term in (2.2) in terms of $\varphi_\pm$ we make use of the Hartree approximation, putting

$$\langle \psi_\pm^\dagger \psi_\pm \psi_\pm \rangle \rightarrow \langle \psi_\pm^\dagger \rangle \langle \psi_\pm \rangle \langle \psi_\pm \rangle. \tag{3.2}$$

We thus obtain the Hartree wave equation for $\varphi_\pm$:

$$i\hbar \frac{\partial \varphi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_p} \nabla^2 \varphi_\pm + f^2 r_p^2 [\varphi_+^* \varphi_+ - \varphi_-^* \varphi_-] \varphi_\pm . \tag{3.3}$$

It can be brought into still another form if we make the same transformation known from the hydrodynamic formulation of Schrödinger's wave equation [6]. For this we put

$$\begin{aligned} n_\pm &= \varphi_\pm^* \varphi_\pm , \\ n_\pm \boldsymbol{v}_\pm &= \mp \frac{i\hbar}{2m_p} [\varphi_\pm^* \nabla \varphi_\pm - \varphi_\pm \nabla \varphi_\pm^*] \end{aligned} \tag{3.4}$$

and obtain from (3.3)

$$\frac{\partial n_\pm}{\partial t} + \operatorname{div}(n_\pm \boldsymbol{v}_\pm) = 0, \tag{3.5}$$

$$\frac{\mathrm{d}\boldsymbol{v}_\pm}{\mathrm{d}t} = \mp \frac{1}{m_p} \operatorname{grad}(V + Q_\pm), \tag{3.6}$$

where

$$\begin{aligned} \frac{\mathrm{d}v_\pm}{\mathrm{d}t} &= \frac{\partial v_\pm}{\partial t} + (\boldsymbol{v}_\pm \cdot \nabla)\, \boldsymbol{v}_\pm , \\ V &= f^2 r_p^2 [n_+ - n_-], \\ Q_\pm &= \mp \frac{\hbar^2}{2m_p} \frac{\nabla^2 \sqrt{n_\pm}}{\sqrt{n_\pm}} . \end{aligned} \tag{3.7}$$

V is the regular potential and $Q_\pm$ the quantum potential.

To obtain the connection between the hydrodynamic formulation and the nonlinear wave equation (3.3) we put

$$\varphi_\pm = A_\pm e^{iS_\pm} \tag{3.8}$$

demanding that

$$A_\pm \geq 0, \qquad 0 \leq S_\pm \leq 2\pi$$

by which $A_\pm$ and $S_\pm$ are uniquely determined, provided $\varphi_\pm \neq 0$. Inserting (3.8) into (3.4) we find

$$n_\pm = A_\pm^2, \qquad \boldsymbol{v}_\pm = \pm \frac{\hbar}{m_p} \operatorname{grad} S_\pm \tag{3.9}$$

from which follows that for regular points

$$\operatorname{curl} \boldsymbol{v}_\pm = 0. \tag{3.10}$$

More generally, one has

$$S_\pm(\boldsymbol{r}, t) = \pm \frac{m_p}{\hbar} \int_{\boldsymbol{r}_0}^{\boldsymbol{r}} \boldsymbol{v}_\pm(\boldsymbol{r}', t) \cdot \mathrm{d}\boldsymbol{r}' + S_\pm^0(t). \tag{3.11}$$

The uniqueness of $S_\pm$ requires that

$$\oint \boldsymbol{v}_\pm \cdot \mathrm{d}\boldsymbol{r} = 0 \tag{3.12}$$

but the uniqueness of $\varphi_\pm$ only requires that

$$\oint \boldsymbol{v}_\pm \cdot \mathrm{d}\boldsymbol{r} = \pm n h/m_p, \qquad n = 0, 1, 2, \dots . \tag{3.13}$$

This last result leads to Helmholtz's vortex theorem for the two superfluids:

$$\frac{\mathrm{d}}{\mathrm{d}t} \oint \boldsymbol{v}_\pm \cdot \mathrm{d}\boldsymbol{r} = 0. \tag{3.14}$$

For the Hartree wave function one finds after a simple calculation, putting $r_0 = 0$, that

$$\varphi_\pm = |\sqrt{n_\pm}| \exp\left[\pm \frac{i m_p}{\hbar} \int_0^{\boldsymbol{r}} \boldsymbol{v}_\pm \cdot \mathrm{d}\boldsymbol{r} + \frac{i}{\hbar} S_\pm^0(t) \right] \tag{3.15}$$

with

$$\begin{aligned} S_\pm^0(t) &= \int_0^t E_\pm(t)\, \mathrm{d}t, \\ E_\pm(t) &= \int (Q_\pm \pm \tfrac{1}{2} m_p \boldsymbol{v}_\pm^2 + V)\, n_\pm \, \mathrm{d}\boldsymbol{r}, \end{aligned} \tag{3.16}$$

where $E_\pm$ is the total energy of the superfluids.

4. The Ground State

The Hartree approximation assumes that in the ground state all the Planckions have zero momentum. With an equal number of positive and negative Planck masses the ground state, therefore, has zero energy.

But this is not the only ground state possible. If one quantized vortex made up from the positive mass superfluid would be present it would lead to an energy larger than the energy of the ground state. However, if at the same time a vortex of the same length and circulation but made up of the negative mass superfluid is produced together with the positive mass vortex, the total energy of this configuration would be the same as for the ground state. This means that a vortex pair could be produced out of the combined positive-negative mass superfluid without the expenditure of any energy by spontaneous symmetry breaking provided, of course, that the angular momenta of both vortices compensate each other. More generally, a three dimensional lattice of positive and negative mass vortex rings could likewise be produced by spontaneous symmetry breaking. The condition for the spontaneous symmetry breaking to occur would be that it leads to a configuration distinguished by its maximum stability against perturbations. A mechanism to cause the spontaneous symmetry breaking is in principle provided by the delta-function force between Planckions of opposite mass, which can lead to the self-acceleration of positive-negative mass dipoles, thereby stirring the superfluid. To determine the different groundstate, consisting of a lattice of positive and negative mass vortex rings, would require to solve a very difficult stability problem. It could be done by a trial wave function for the vortex lattice, using a product of wave functions (3.15) for one vortex. Because it would be a very complicated problem to solve, we will instead use a hydrodynamic analogy borrowed from viscous fluid dynamics to determine the lattice structure.

For a linear vortex, with the maximum velocity $v = c$ reached at $r = r_p$, the velocity is that of a potential vortex, with v_p the azimuthal velocity component in cylindrical coordinates given by

$$v_\varphi = c(r_p/r). \tag{4.1}$$

As can be seen from (1.3) and (3.13), the smallest vortex radius r_0 for the different quantum numbers n is

$$r_0 = n\, r_p. \tag{4.2}$$

Each vortex can move freely in an environment where n_+ and n_- are uniform, except for the core of the vortex itself. Equation (4.1) also applies to ring vortices with a ring radius $R \gg r_p$. In the lowest state of a ring vortex one has $n = 1$. It is reasonable to assume that the spacing of the ring vortices is of the same order as R. If the ratio R/r_p becomes too small, the vortex rings may disturb each other destroying their individual integrity, but if R/r_p becomes too large, the vortex rings may become unstable through the force exerted on them by the surrounding fluid. In viscous fluid dynamics, the drag on an object has a sharp minimum for a Reynold's number in between 10^5 and 10^6. An infinite cylinder of radius r, for example, [7], suffers a minimum drag if the Reynold's number is

$$\mathrm{Re} = v\, r/\nu \simeq 250\,000, \tag{4.3}$$

where ν is the kinematic viscosity. The Reynold's number is the ratio in between the inertial forces $\varrho\, v^2/r$ and the viscous forces $\varrho\, \nu\, v/r^2$. The viscous forces tend to dampen out perturbations which for $\nu = 0$ would grow beyond all limits. In a superfluid $\nu = 0$, but there the quantum force, which is the gradient of the quantum potential, counteracts the inertial force in trying to keep the vortices in their fixed quantum state. Outside the vortex core $n \simeq$ const and $Q \simeq 0$, but in the vicinity of the core one has

$$|Q| \simeq \frac{\hbar^2}{2 m_p r_p^2} \tag{4.4}$$

with a quantum force per unit volume

$$|f_Q| \simeq \frac{\hbar^2}{2 m_p r_p^6}. \tag{4.5}$$

The inertial force density in the core is

$$|f_i| \simeq \varrho\, c^2/r_p = m_p\, c^2/2 r_p^4 \tag{4.6}$$

and the ratio of both

$$|f_i|/|f_Q| = 1. \tag{4.7}$$

If we call this ratio the quantum Reynolds number, Re^Q, we have in the vortex core

$$\mathrm{Re}^Q = v\, r/\nu_Q = c\, r_p/\nu_Q = 1, \tag{4.8}$$

where $\nu_Q = c\, r_p$, which we may call the quantum viscosity. Applied to a vortex as a whole we must average ν_Q over the entire vortex, not just take its value at $r = r_p$. This average is

$$\overline{\nu_Q} \simeq c\, r_p (r_p/R)^2. \tag{4.9}$$

The average value for Re^Q, therefore, is

$$\overline{\mathrm{Re}^Q} \simeq (R/r_p)^2. \tag{4.10}$$

We then assume that the smallest disturbance of the ring occurs for the same quantum Reynolds number as for the Reynolds number in viscous fluid dynamics, minimizing the drag acting on a cylinder with the same radius R as the vortex ring radius. Therefore, putting into (4.10) $\mathrm{Re}^{\varrho} \simeq 250\,000$ and solving for R we find

$$R \simeq 500\, r_p . \tag{4.11}$$

This is a length which is whithin one order of magnitude equal to the poorly determined grand unification scale for which in the literature values $10^3 - 10^4$ are given. Our estimate of R, is of course at least as uncertain.

The lattice structure gives a plausible explanation for the phenomenon of charge quantization by considering two vortex rings belonging to different circulation quantum numbers n, with their core radius given by (4.2), and with their ring radius R unchanged. Since the Planckions are attached to the vortex core, the Planckion charge of a ring is proportional to its surface, which is proportional to the circulation quantum number n.

5. Longitudinal Waves

Let us assume a situation where $n_- = \text{const}$, but where n_+ and v_+ can vary in space and time. A disturbance of this kind leads to compressional longitudinal waves in the substratum. As we will see, the consideration of these waves will give us a means to determine the value of the coupling constant f entering our fundamental nonlinear equation (2.2).

With $\boldsymbol{v}_+ \equiv \boldsymbol{v}$, $n_+ m_p = n m_p = \varrho$, and $V = f^2 r_p^2 n = f^2 r_p^2 \varrho / m_p$, small amplitude waves are determined by the equation

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{f^2 r_p^2}{m_p^2} \nabla \varrho . \tag{5.1}$$

Comparing this result with the equation for small amplitude compressional sound waves

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{1}{\varrho} \nabla p , \tag{5.2}$$

where ϱ is the pressure in the fluid, we find that

$$p = \frac{f^2 r_p^2}{2 m_p^2} \varrho^2 . \tag{5.3}$$

In this way we have obtained an equation of state for the Planckion fluid, which formally is the same as for an ideal gas with a specific heat ratio $\gamma = 2$. Therefore, it has a velocity of sound c_s given by

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\varrho} = \gamma \frac{p}{\varrho} = 2 \frac{p}{\varrho} = \frac{f^2 r_p^2}{m_p^2} \varrho . \tag{5.4}$$

Because $\varrho = n m_p = (1/2)\, m_p / r_p^3$, it follows that

$$c_s^2 / c^2 = (1/2) f^2 / \hbar c . \tag{5.5}$$

Setting $c_s = c$ results in $f^2 = 2\hbar c = 2 G m_p^2$.

The zero point fluctuations of the Planckions attached to the vortex rings become the source of virtual phonons, which can influence the compression waves in the range in between r_p and R. According to the uncertainty principle the quantum energy fluctuation $\Delta\varepsilon$ of a Planckion confined in a vortex filament is:

$$\Delta\varepsilon \simeq (1/2)\, m_p c^2 \simeq \hbar c / 2 r_p . \tag{5.6}$$

Within the volume $2 r_p^3$ this leads to an energy density given by

$$\Delta\varepsilon / 2 r_p^3 \simeq \hbar c / 4 r_p^4 . \tag{5.7}$$

We compare (5.7) with the energy density of the gravitational field for a Planck mass. This field is

$$F = \frac{\sqrt{G}\, m_p}{r^2} = \frac{\sqrt{\hbar c}}{r^2} = \frac{f/\sqrt{2}}{r^2} . \tag{5.8}$$

At a distance $r = r_p$ it has the energy density

$$F^2 / 8\pi = \hbar c / 8\pi r_p^4 . \tag{5.9}$$

Apart from the factor $1/2\pi$ this is the same energy density as the one obtained from the zero point fluctuations. And up the factor $1/\sqrt{2}$, the "charge" f of a Planckion is, therefore, equal to the gravitational coupling constant $\sqrt{G}\, m_p$. This result permits to explain the gravitational charge of the Planck masses in a purely mechanistic way. The quantum fluctuations of the Planck masses produce virtual longitudinal compression waves in the substratum and which are the cause for an attractive potential obeying Newton's law. We remark that already in classical fluid dynamics an attractive inverse square force law is set up in between two pulsating spheres immersed in an incompressible fluid [8–10].

The number of Planckion charges along a vortex ring of radius R is $\sim 2\pi R / 2 r_p$, and the volume occupied by the rings in a vortex ring lattice is of the order $(2R)^3$, containing $(2R)^3 / 2 r_p^3$ Planckions of one sign. The effective Planckion charge per unit volume is, therefore, reduced from $1/2 r_p^3$ by the factor $(2\pi R/2r_p)$ $(2r_p^3/(2R)^3) \simeq (r_p/R)^2$. The Planckion charges are setting up a field determined by a Poisson-type equation:

$$\operatorname{div} \boldsymbol{F} = 4\pi f (r_p/R)^2 n', \tag{5.10}$$

where $n' \ll n = 1/2r_p^3$. The force is here repulsive, because an increased ether density displaces and thereby dilutes the vortex filaments embedded in it. Instead of (5.1) the equation of motion then becomes

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{f^2 r_p^2}{m_p} \nabla n' + \frac{f}{m_p} \boldsymbol{F}. \tag{5.11}$$

With the continuity equation

$$\partial n'/\partial t + n \operatorname{div} \boldsymbol{v} = 0 \tag{5.12}$$

and putting $\theta = \operatorname{div} \boldsymbol{v}$, we obtain a wave equation for θ:

$$-(1/c^2)\,\partial^2\theta/\partial t^2 + \nabla^2\theta - (\omega_0^2/c^2)\,\theta = 0, \tag{5.13}$$

where

$$\omega_0^2 = 4\pi(c/R)^2. \tag{5.14}$$

Equation (5.13) has the dispersion relation

$$\omega^2 = c^2 k^2 + \omega_0^2. \tag{5.15}$$

The phase velocity of the compression waves is

$$v_{ph} = \frac{\omega}{k} = \frac{c}{\sqrt{1-(\omega_0/\omega)^2}} \tag{5.16}$$

and the group velocity

$$v_g = \frac{d\omega}{dk} = c\sqrt{1-(\omega_0/\omega)^2} \tag{5.17}$$

such that $v_{ph}\, v_g = c^2$.

We, therefore, see that the longitudinal compression waves have a superluminal phase velocity going to infinity at a wave length which is by order of magnitude equal to the vortex ring radius R, and have a cut-off for wave lengths larger than R.

6. Transverse Waves

In addition to the longitudinal compression waves there are transverse waves. The way how they are transmitted through a vortex lattice is shown in Figure 1. Whereas, the longitudinal waves have an upper cut-off at a wave length $\lambda \sim R$, the transverse waves have lower cut-off at the same wave length. These waves were first analyzed by Thomson [2]. Before

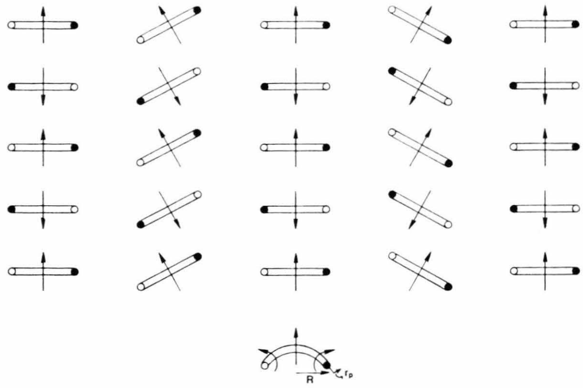

Fig. 1. Vortex wave: Shown is one wavelength of a vortex wave moving from the left to the right.

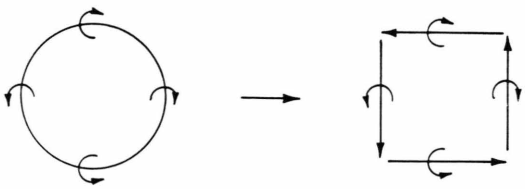

Fig. 2. Transformation of one vortex ring into four spinning tops.

giving a formal derivation, we would like to explain the underlying physics. In Fig. 2, the transformation of one vortex ring into 4 linear vortices, with pairs of parallel vortices rotating in opposite directions, is shown. If the rotating linear vortices are replaced by solid masses we get 4 spinning tops. Next, consider the transformation of the entire vortex lattice of the substratum, consisting of vortex rings oriented in all three spatial directions, into such spinning tops. By making this transformation, we end up in what is known as Kelvin's gyrostatic ether model. If the tops are coupled with each other, this ether model leads to transverse waves simulating electromagnetic waves.

As it is known from classical mechanics, a top resists a change in its direction. For small amplitude waves, and in the limit where the size of the tops goes to zero with an infinite number of tops filling space, this ether model can be described by a skew symmetric stress tensor σ_{ik} for which

$$\left.\begin{aligned} \sigma_{xy} &= -\sigma_{yx} = (k/2)\,\varphi_z \\ \sigma_{yz} &= -\sigma_{zy} = (k/2)\,\varphi_x \\ \sigma_{zx} &= -\sigma_{xz} = (k/2)\,\varphi_y \end{aligned}\right\} \tag{6.1}$$

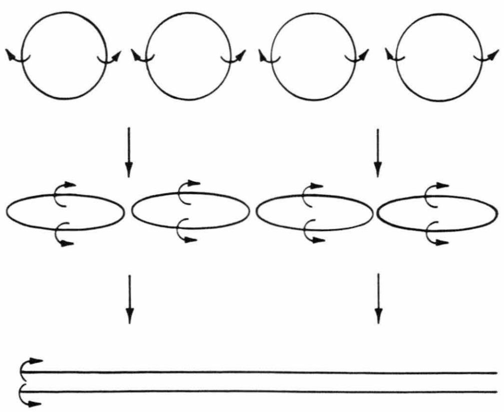

Fig. 3. Transformation of the vortex lattice into the vortex sponge by deformation and coalescence of a chain of vortex rings into a pair of vortex filaments.

The vector φ is an angle by which the ether is twisted, such that $k\,\varphi$ is the moment of this twist per unit mass, with k a twist modulus. The small amplitude equation of motion of this ether, with $\boldsymbol{u}$ the ether velocity and ϱ the ether density, is

$$\varrho\,\frac{\partial \boldsymbol{u}}{\partial t} = -\frac{k}{2}\,\mathrm{curl}\,\boldsymbol{\varphi}\,. \tag{6.2}$$

Equation (6.2) must be supplemented by the kinematic relation

$$\frac{\partial \boldsymbol{\varphi}}{\partial t} = \frac{1}{2}\,\mathrm{curl}\,\boldsymbol{u}\,. \tag{6.3}$$

Assuming that $\mathrm{div}\,\boldsymbol{u} = \mathrm{div}\,\boldsymbol{\varphi} = 0$, elimination of $\boldsymbol{\varphi}$ from (6.2) and (6.3) leads to the wave equation

$$-(1/c^2)\,\partial^2\boldsymbol{u}/\partial t^2 + \nabla^2\boldsymbol{u} = 0\,, \tag{6.4}$$

where we have set $c^2 = k/4\,\varrho$. Eqs. (6.2) and (6.3), together with $\mathrm{div}\,\boldsymbol{u} = \mathrm{div}\,\boldsymbol{\varphi} = 0$ are known as MacCullagh's ether wave equations [11]. They have the same form as Maxwell's equations in free space, if one sets $\boldsymbol{u} = \boldsymbol{E}$ and $\boldsymbol{\varphi} = -(1/2c)\,\boldsymbol{H}$.

We had to choose for the twist constant $k = 4\,\varrho\,c^2$ to make these waves propagate with the velocity of light. A simple argument justifies this choice. In Figure 3, we deform the vortex rings as shown into elliptic shapes until the ellipses can be linked up into long linear vortex filaments with the bridges between ellipses cancelling out. For the vortex rings to permit this deformation they must all have the same sense of rotation, because only then can the elements touching each other cancel out. Since it is always possible to subdivide the vortex lattice into such families of equally rotating vortices, the described change into linear vortex filaments transforms the vortex lattice into what is called a vortex sponge. Again, waves transmitted through such a vortex sponge have the same property as those derived from Maxwell's equations [12]. Most simply we can envision transverse waves propagating along the vortex filaments. With their velocity c reached at their core radius $r = r_{\mathrm{p}}$, the stress in the vortex filament is $\sigma = \varrho\,c^2$. The wave velocity propagated along a filament of density ϱ and under the stress σ is $\sqrt{\sigma/\varrho}$ and which just turns out to be c. We, therefore, have $k = 4\sigma$.

Finally, we present a greatly simplified version of Kelvin's formal derivation. Let $\boldsymbol{v} = \{v_x, v_y, v_z\}$ be the undisturbed velocity of the ether, and $\boldsymbol{u} = \{u_x, u_y, u_z\}$ a small superimposed velocity disturbance, but let us consider only those solutions for which $\mathrm{div}\,\boldsymbol{v} = \mathrm{div}\,\boldsymbol{u} = 0$. This means, let us only consider transverse waves with no change in density, which according to (5.3) implies that there are no pressure disturbances going along with these waves. In the undisturbed state the ether velocity field is given by the vortex lattice. To make the problem amenable for an analytic treatment, we must let the scale R of the vortex lattice go to zero. In our situation though $R \to r_{\mathrm{p}}$, rather than $R \to 0$.

The x-component of the equation of motion for a disturbance $\boldsymbol{u}$ is given by

$$\begin{aligned}\frac{\partial v_x}{\partial t} + \frac{\partial u_x}{\partial t} = &-(v_x + u_x)\,\frac{\partial (v_x + u_x)}{\partial x}\\ &-(v_y + u_y)\,\frac{\partial (v_x + u_x)}{\partial y}\\ &-(v_z + u_z)\,\frac{\partial (v_x + u_x)}{\partial z} - \frac{1}{\varrho}\,\frac{\partial p}{\partial x}\,.\end{aligned} \tag{6.5}$$

From the continuity equation $\mathrm{div}\,\boldsymbol{v} = 0$ we have

$$-v_x\,\frac{\partial v_x}{\partial x} - v_x\,\frac{\partial v_y}{\partial y} - v_x\,\frac{\partial v_z}{\partial z} = 0\,. \tag{6.6}$$

Adding (6.6) to (6.5) and taking the y-z average we find

$$\frac{\partial u_x}{\partial t} = -\frac{\partial(\overline{v_y v_x})}{\partial y} - \frac{\partial(\overline{v_z v_x})}{\partial z} \tag{6.7a}$$

and similar, by taking the x-z and x-y averages:

$$\frac{\partial u_y}{\partial t} = -\frac{\partial(\overline{v_x v_y})}{\partial x} - \frac{\partial(\overline{v_z v_y})}{\partial z} \tag{6.7b}$$

$$\frac{\partial u_z}{\partial t} = -\frac{\partial(\overline{v_x v_z})}{\partial x} - \frac{\partial(\overline{v_y v_z})}{\partial y}\,. \tag{6.7c}$$

We note that $\overline{v_x v_y} \neq \overline{v_y v_x}$ because for $\overline{v_x v_y}$ we took the x-z average, whereas for $\overline{v_y v_x}$ the y-z average was taken. In general $\overline{v_i v_k} \neq \overline{v_k v_i}$. With the condition $\operatorname{div} \boldsymbol{u} = 0$ we obtain from (6.7a–c) that

$$\overline{v_i v_k} = - \overline{v_k v_i}\,. \tag{6.8}$$

Taking the x-component of the equation of motion, multiplying it by v_y and then taking the y-z average, and the y-component multiplied by v_x taking the y-z average, finally subtracting the first from the second equation, we find

$$\frac{\partial}{\partial t}(\overline{v_x v_y}) = - v^2 \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right), \tag{6.9}$$

where $v^2 = \overline{v_x^2} = \overline{v_y^2}$ is the average microvelocity of the vortex lattice. Putting $\varphi_z = - \overline{v_x v_y}/2 v^2$, (6.9) is just the z-component of

$$\frac{\partial \boldsymbol{\varphi}}{\partial t} = \frac{1}{2} \operatorname{curl} \boldsymbol{u}\,, \tag{6.10}$$

where $\varphi_x = - \overline{v_y v_z}/2 v^2$, $\varphi_y = - \overline{v_z v_x}/2 v^2$.

Equations (6.7a–c) then take the form

$$\partial \boldsymbol{u}/\partial t = - 2 v^2 \operatorname{curl} \boldsymbol{\varphi}\,. \tag{6.11}$$

Equations (6.11) and (6.10) are equivalent with MacCullagh's equations (6.2) and (6.3). Elimination of $\boldsymbol{\varphi}$ from (6.10) and (6.11) gives

$$- (1/v^2)\, \partial^2 \boldsymbol{u}/\partial t^2 + \nabla^2 \boldsymbol{u} = 0\,. \tag{6.12}$$

In the collapsed vortex lattice making $R \to r_p$, one should have for the microvelocity $v^2 = c^2$. Equation (6.12) would then describe a transverse wave propagating with the velocity of light c. In reality though, $R \sim 10^3 r_p$. This means that Maxwell's equations describing electromagnetic waves would break down at energies corresponding to the scale R. This energy is of the order $\sim 10^{16}$ GeV.

The theory for the vortex waves is valid in the limit $R \to r_p$. In the vortex lattice, where the vortex ring radius is $R \gg r_p$, and the separation between the vortex rings of the same order of magnitude, the coupling of the vortices is achieved by the longitudinal compression waves in the wave length range $r_p \lesssim \lambda \lesssim R$. Since these compression waves propagate with the velocity of light, the vortex waves should have the same propagation velocity. In the limit $R \to r_p$, where the vortex cores touch each other, the coupling is provided by the "pressure and thrust", of an incompressible fluid, simulating the coupling caused by the compression waves.

It is now easy to show how one obtains Maxwell's equations in the presence of charges. By virtue of Gauss' theorem for a field produced by fluctuating Planckions, and by putting $\boldsymbol{E} = \boldsymbol{u}$, MacCullagh's equation $\operatorname{div} \boldsymbol{u} = 0$, becomes

$$\operatorname{div} \boldsymbol{E} = 4\pi \varrho_e\,, \tag{6.13}$$

where ϱ_e is the electric charge density. Then, if an electric current density vector $\boldsymbol{j}_e$ is introduced, with ϱ_e and $\boldsymbol{j}_e$ satisfying an equation of charge conservation

$$\partial \varrho_e/\partial t + \operatorname{div} \boldsymbol{j}_e = 0 \tag{6.14}$$

a term must be added to MacCullagh's equation (6.2). With $\boldsymbol{\varphi} = - (1/2c) \boldsymbol{H}$ it, thereby, becomes Maxwell's equation

$$\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi}{c} \boldsymbol{j}_e = \operatorname{curl} \boldsymbol{H}\,. \tag{6.15}$$

MacCullagh's equation (6.3) and which is purely kinematic, is unchanged and becomes

$$- \frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} = \operatorname{curl} \boldsymbol{E}\,. \tag{6.16}$$

Finally, $\operatorname{div} \boldsymbol{\varphi} = 0$ becomes

$$\operatorname{div} \boldsymbol{H} = 0\,. \tag{6.17}$$

We may compute the value for the stiffness constant k in MacCullagh's equation (6.2). With $\varrho = m_p/2 r_p^3$ one there finds for the stiffness constant

$$k = 4 \varrho c^2 = 2 m_p c^2/r_p^3 = 2\hbar c/r_p^4 = 2 G m_p^2/r_p^4\,. \tag{6.18}$$

It turns out to be extremely large and explains why the linear Maxwell equations are so well reproduced by the linearized mechanical equations of the ether models by MacCullagh and Thomson.

Part II

7. Lorentz and Gauge Invariance

As we have shown, the dynamic behavior of the superfluid Planckion ether for distances large against $R \sim 10^3 r_p$, can be described by a set of equations which in a system at rest with the ether are the same as Maxwell's equations. Lorentz invariance, however, means that these equations would have to remain the same in a system in uniform motion against the ether. We will show below that fermions can be understood as bound states made up from the positive and negative masses of the Planckion ether. Elementary particles are, therefore, held together by the kind of forces which in a system at rest with the ether can be described by Maxwell's equations, or in the language of quantum field theory, bound together by vector gauge bosons. If bodies in a state of internal equilibrium are held together by electromagnetic forces, Lorentz invariance can be understood as a dynamic symmetry for the following reason: Let us consider a body which initially is at rest in the substratum, and thereafter accelerated to a constant velocity $\boldsymbol{u}$ against the substratum. If at rest in the substratum, but also in a state of internal equilibrium, the scalar and vector potentials within the body are given by Maxwell's equation

$$\nabla^2 \Phi = -4\pi\varrho, \quad \nabla^2 \boldsymbol{A} = -(4\pi/c)\boldsymbol{j} \tag{7.1}$$

with the gauge condition

$$\operatorname{div} \boldsymbol{A} = 0. \tag{7.2}$$

The electric charge and current densities, ϱ and $\boldsymbol{j}$, have their source in the body. After the body has been accelerated to the constant velocity $\boldsymbol{u}$ against the substratum, and after it has assumed a new equilibrium state, the static Galilei transformed equations for Φ and $\boldsymbol{A}$ in a comoving reference system are needed. General Galilei transformed equations for the electromagnetic potentials and the gauge equation have been given by Wilhelm [13]. More convenient for the problem in question are equations for the transformed potentials, if the gauge condition is made Galilei invariant. This is possible because of the freedom one has in choosing the gauge. A Galilei invariant choice of the gauge also makes the continuity equation Galilei invariant, a desired property of this equation is applied to a body in a comoving frame of reference. The equations for the potentials under a Galilei transformation

$$\boldsymbol{r}' = \boldsymbol{r} - \boldsymbol{u}t, \quad t' = t \tag{7.3}$$

then are:

$$\begin{aligned} \left[-\frac{1}{c^2}\left(\frac{\partial}{\partial t} - \boldsymbol{u}\cdot\nabla'\right)^2 + \nabla'^2\right]\Phi' &= -4\pi\varrho', \\ \left[-\frac{1}{c^2}\left(\frac{\partial}{\partial t} - \boldsymbol{u}\cdot\nabla'\right)^2 + \nabla'^2\right]\boldsymbol{A}' &= -(4\pi/c)\boldsymbol{j}', \end{aligned} \tag{7.4}$$

where ∇' is the Galilei transformed Nabla operator. The Galilei invariance of the continuity equation $\partial\varrho'/\partial t + \operatorname{div}\boldsymbol{j}' = 0$ then immediately follows from (7.4) provided the Lorentz gauge is made Galilei invariant:

$$\frac{1}{c}\frac{\partial\Phi'}{\partial t} + \operatorname{div}\boldsymbol{A}' = 0. \tag{7.5}$$

After the body has settled down to a new equilibrium state, the static equations for the potentials Φ' and $\boldsymbol{A}'$ are obtained putting $\partial/\partial t = 0$:

$$\begin{aligned} \left[\left(1-\frac{u^2}{c^2}\right)\nabla_{\parallel}'^2 + \nabla_{\perp}'^2\right]\Phi' &= -4\pi\varrho', \\ \left[\left(1-\frac{u^2}{c^2}\right)\nabla_{\parallel}'^2 + \nabla_{\perp}'^2\right]\boldsymbol{A}' &= -(4\pi/c)\boldsymbol{j}', \end{aligned} \tag{7.6}$$

and

$$\operatorname{div}\boldsymbol{A}' = 0, \tag{7.7}$$

where $\nabla_{\parallel}$ and $\nabla_{\perp}$ are the components of the Nabla operator parallel and perpendicular to the direction of $\boldsymbol{u}$.

Comparing (7.6–7.7) with (7.1–7.2), one immediately sees that (7.6–7.7) as (7.1–7.2) and are the same if one puts everywhere, including in the sources,

$$\mathrm{d}\boldsymbol{r}_{\parallel}' = \sqrt{1-u^2/c^2}\,\mathrm{d}\boldsymbol{r}_{\parallel}, \quad \mathrm{d}\boldsymbol{r}_{\perp}' = \mathrm{d}\boldsymbol{r}_{\perp} \tag{7.8}$$

which makes

$$\left(1-\frac{u^2}{c^2}\right)\nabla_{\parallel}'^2 + \nabla_{\perp}'^2 = \nabla^2. \tag{7.9}$$

Therefore, $\Phi' = \Phi$ and $\boldsymbol{A}' = \boldsymbol{A}$, provided $\varrho'(\boldsymbol{r}') = \varrho(\boldsymbol{r})$, $\boldsymbol{j}'(\boldsymbol{r}') = \boldsymbol{j}(\boldsymbol{r})$, implying a uniform contraction of the sources by the Fitzgerald-Lorentz factor $\sqrt{1-u^2/c^2}$. The continuity and and gauge condition are unaffected by this change.

If all the fundamental interactions behave like the electromagnetic interactions, all clocks should behave like light clocks, and in considering the combined effect of the Lorentz contraction and anisotropic light propagation in a moving frame makes a light clock move slower by the same factor $\sqrt{1-u^2/c^2}$, as in special relativity. The Lorentz contraction alone is, therefore, sufficient to derive the Lorentz transformations as a dynamic symmetry for objects in a state of internal equilibrium [3].

The interaction in between the vortices, which according to our model is communicated by compressional waves in the wavelength range $r_{\mathrm{p}} < \lambda < R$, is also subject to Lorentz invariance as a dynamic symmetry principle, because the wave equation for the compressional wave has the same form as the wave equation for the scalar component Φ of the electromagnetic potential.

From the Lorentz transformations follows the relativistic addition theorem of velocities, and from the velocity addition theorem the relativistic variation of the mass with the velocity, $m = m_0/\sqrt{1-u^2/c^2}$. In conjunction with Newton's equation of motion $(\mathrm{d}/\mathrm{d}t)(m\,\boldsymbol{v}) =$ force, then follows the relativistic expression for the energy E. If all energy is electromagnetic in origin, one has $E = E_0/\sqrt{1-u^2/c^2}$, where E_0 is the electromagnetic energy in the ether rest frame. From the Lorentz transformations and the relativistic expression for the energy follows the relativistically invariant Hamilton function. If translated into quantum mechanics it leads to the relativistically invariant Hamiltonian and ensures that the contraction effect does not change the pressure by the zero point energy*.

We should add that the change in mass with velocity does not apply to the Planckions because they obey a nonrelativistic law of motion. It is for this reason that Lorentz invariance as a dynamic symmetry would break down at the Planck scale.

With Maxwell's equations being invariant under a Lorentz transformation seen as a dynamic symmetry due to a true deformation of measuring rods and clocks, gauge invariance is automatically established as well. Normally, current conservation is derived from gauge invariance through Noether's theorem. Here the sequence in the derivation is reversed. Because the Planckion fluid is nonrelativistic, it obeys a nonrelativistic continuity equation, which for the Planckions means conservation of the Planckion charges. It is from this charge conservation law that gauge invariance is derived, not the other way around.

If Lorentz invariance is a dynamic symmetry it will also act as a dynamic selection principle, selecting from all configurations possible and held together by vector gauge bosons, those which are in a stable equilibrium. We will show below that fermions might be made up from objects having the dimension R. We may, therefore, say that fermions with a lifetime

$$\tau \gg \tau_0 \sim R/c \sim 10^{-41}\,\mathrm{sec} \tag{7.10}$$

are sufficiently stable to satisfy Lorentz invariance as a dynamic selection principle. The dynamic selection principle would exclude all those theories which are nonrenormalizable, because these theories depend on a cut-off at the Compton wave length of some elementary particle, rather than a cut-off at $R \sim 10^{-30}$ cm.

If Lorentz invariance must be understood as a dynamic symmetry, valid only within the adiabatic approximation for times large compared to the time τ_0, it would mean that theorems like the CPT theorem, or the spin-statistics theorem derived under the assumption that special relativity is exactly fulfilled, could be violated for times shorter than this time.

8. The Energy Spectrum of the Substratum

The energy spectrum of the substratum has two branches, a positive branch for the superfluid of the positive mass Planckions, and a negative one for the negative mass Planckions. Both branches are symmetric with respect to the wave number k. One then has to distinguish between two wave number regions, the first one for $0 < k < 1/R$ and the second one for $1/R < k < k_\mathrm{p}$, where $k_\mathrm{p} = m_\mathrm{p}\,c/\hbar$ is the Planck wave number. In the first region, it is a spectrum of zero rest mass vector gauge bosons for which

$$\varepsilon_\pm^{(1)}(k) = \pm\,\hbar\,c\,k\,. \tag{8.1}$$

In the second region, dominated by longitudinal waves, the spectrum has the typical phonon-roton structure known from the theory of liquid Helium [14]:

$$\varepsilon_\pm^{(2)}(k) = \pm\,\hbar^2 k^2/2\,m_\mathrm{p}\,S(k)\,, \tag{8.2}$$

where $S(k)$ is the liquid structure function. The roton part can be approximated by an expansion in the vicinity of a minimum in $|\varepsilon_\pm(k)|$ positioned at k_0:

$$\varepsilon_\pm^{(2)}(k) \simeq \pm\left(\Delta + \frac{\hbar^2(\boldsymbol{k}-\boldsymbol{k}_0)^2}{2m^*}\right), \tag{8.3}$$

where m^* is an effective roton mass and Δ an energy gap. From the data of liquid Helium, somewhat resembling our situation, one would have (by replacing the Helium mass with m_p and the Debye-wave number with k_p): $k_0 \simeq 0.7\,k_\mathrm{p}$, $m^* \simeq 0.16\,m_\mathrm{p}$, $\Delta \simeq 0.5\,m_\mathrm{p}\,c^2$. This

* The pressure by the zero point energy results from the uncertainty principle, which for electromagnetic energy $m\,c^2$ is $\Delta m\,c^2 \cdot \Delta r \geq \hbar\,c$. If $\Delta r_\parallel$ changes into $\sqrt{1-u^2/c^2}\,\Delta r_\parallel = \Delta r'_\parallel$, it changes $\Delta m\,c^2$ into $(\Delta m\,c^2)/\sqrt{1-u^2/c^2} = (\Delta m\,c^2)'$, and, therefore, keeps $(\Delta m\,c^2)' \cdot \Delta r'_\parallel = \Delta m\,c^2 \cdot \Delta r_\parallel$ invariant under this change of scale. As a result, the zero point energy pressure remains the same.

comparison suggests that the roton part of the spectrum must be close to the cut-off at $k = k_p$.

In the immediate vicinity of $k = k_p$ the spectrum is likely to resemble the form known from a solid state near the Debye wave number k_p:

$$\varepsilon_{\pm}^{(2)}(k) \simeq \pm (2/\pi)\, \hbar c k_p \sin[(\pi/2)(k/k_p)]. \tag{8.4}$$

The transition from region 1 to 2 is more difficult to guess. More important is that for $k < 1/R$ there is likely to be a bound state due to a resonance near the frequency of the vortex circular velocity. At $r \sim R$, it is of the order

$$\omega_v \sim c r_p/R^2. \tag{8.5}$$

Associated with this resonance is a "vorton" energy of the order

$$\varepsilon_v = \hbar \omega_v \sim m_p c^2 (r_p/R)^2. \tag{8.6}$$

For a single vortex ring and small amplitude oscillations, the exact value of this resonance frequency has been computed by Thomson [15], who showed that it occurs for an elliptic deformation of the ring. Its value is

$$\omega_v \simeq 2\sqrt{3}(r_p c/R^2)\ln(8R/r_p) \tag{8.7}$$

and hence

$$\varepsilon_v \simeq 2\sqrt{3}\, m_p c^2 (r_p/R)^2 \ln(8R/r_p). \tag{8.8}$$

It was, furthermore, shown by Thomson that for linked vortices the resonance frequency becomes smaller. Hicks [16] estimates that for a lattice of vortices the frequency is approximately equal to the value by (8.5).

It is a very different question how the intermediate massive vector bosons fit into this picture. One guess is that they may be solitons of the kind conjectured by Hartley [17] for the gyrostatic ether. The stiffness of such an ether goes down with time resp. increasing amplitude. As Hartley has shown, the existence of soliton solutions are suggested by making a comparison with Riemann's general finite amplitude solution for sound waves. For an ideal gas this solution is [18]:

$$v = F[x - (\pm c_0 + \tfrac{1}{2}(\gamma + 1)v)t], \tag{8.9}$$

where F is an arbitrary function, v the fluid velocity in the wave propagating in the x-direction, c_0 the velocity of sound in the undisturbed medium, and γ the specific heat ratio. Equation (8.9) has soliton-type solutions provided $\gamma = -1$. Translated into an equation of state, relating the pressure with the density, this would mean that

$$p = -\text{const}/\varrho \tag{8.10}$$

and by which the velocity of sound $\sqrt{\gamma p/\varrho}$ would be proportional to ϱ^{-1}. The velocity of sound would there decrease with increasing density and hence wave amplitude, very similar to what is expected to happen for a wave in the gyrostatic ether model.

What remains is the difficult question regarding the stability of the vortex lattice. In the frame of classical fluid dynamics, it was shown by William Thomson [19] that stable configurations are possible, but no corresponding analysis has been made for quantum fluids. Nevertheless, the empirically established fact that lattices of vortex filaments in superfluids and superconductors are stable, supports the hypothesis for the proposed vortex structure of the superfluid quantum ether.

9. Dirac Spinors

As we will now show, the hypothesis that the substratum consists of positive and negative masses, which was made possible by the dynamic interpretation of special relativity, leads to Dirac spinors. Already Schrödinger [20] had noticed that the spin can be explained by the negative energy (and, hence, mass) states in Dirac's equation. As he showed these states lead to a microscopic motion which he called "Zitterbewegung," and which generates the angular momentum responsible for the spin. Reversing Schrödinger's reasoning it was shown by Hönl and Papapetrou [21] that the superposition of a mass pole with a positive-negative mass dipole (pole-dipole particle) leads to Dirac type equations. Furthermore, it was shown by Bopp [22] that the negative masses can be accounted for within the framework of Hamiltonian mechanics if one considers Lagrange functions of the form $L(q_k, \dot{q}_k, \ddot{q}_k)$. The Euler Lagrange equations of the variational principle

$$\delta \int L(q_k, \dot{q}_k, \ddot{q}_k)\, dt = 0 \tag{9.1}$$

then lead to a set of two canonical equations, one for the macrovariables describing the system as a whole, and one for microvariables describing the Zitterbewegung-type degrees of freedom. It was subsequently shown by Hönl [23] that Bopp's theory is to a large extent equivalent to the pole-dipole particle model.

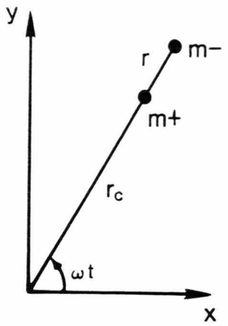

Fig. 4. Pole-dipole particle configuration.

Bopp, therefore, also obtained Hamiltonians of the Dirac type.

Following Hönl and Papapetrou, we first analyze the simple classical mechanical two body pole-dipole model shown in Figure 4. It consists of a positive mass m^+ and a negative mass m^-. In a two body problem with both masses positive and with an attractive force in between, the two bodies can execute a circular motion around their center of mass. In case one of the masses is negative, but with both together having a positive mass pole $m_0 = m^+ - |m^-|$, the circular motion persists, except that the center of mass is no more between the masses, even though it is still located on the line connecting m^+ and m^-. As a consequence, the pole-dipole particle executes a rotational motion and which causes the spin. This motion has the same property as the "Zitterbewegung" derived by Schrödinger.

If $|m^+| > |m^-|$, the distance of m^- from the center of mass is larger than for m^+, and we assume that m^+ is at a distance r_c, with m^- at a distance $r_c + r$. Furthermore, if $m_0 \ll m^+ \simeq |m^-|$, one has $r \ll r_c$. Defining $\gamma_+ = (1 - v_+^2/c^2)^{-1/2}$, with $v_+ = r_c\omega$, where ω is the angular velocity around the center of mass, and $\gamma_- = (1 - v_-^2/c^2)^{-1/2}$, with $v_- = (r_c + r)\omega$, momentum conservation leads to

$$m^+ \gamma_+ r_c = |m^-| \gamma_- (r_c + r). \tag{9.2}$$

For $r \ll r_c$ we can expand:

$$\gamma_- = \gamma\left(1 + \frac{r_c r \omega^2 \gamma^2}{c^2} + \dots\right), \tag{9.3}$$

putting henceforth $\gamma_+ \equiv \gamma$.

For the mass dipole moment we have

$$p = m^+ r \simeq |m^-| r = \frac{m^+ \gamma - |m^-| \gamma_-}{\gamma_-} r_c . \tag{9.4}$$

With the help of (9.3) and for $\gamma \gg 1$ we find

$$r_c \simeq p \gamma^2 / m_0 . \tag{9.5}$$

For the energy we find

$$E/c^2 = m = m^+ \gamma - |m^-| \gamma_- \simeq p\gamma / r_c \tag{9.6}$$

and finally, for the angular momentum (putting $\omega r_c \simeq c$):

$$J = [m^+ \gamma r_c^2 - |m^-| \gamma_- (r_c + r)^2]\omega \simeq - p\gamma c \simeq - m c r_c . \tag{9.7}$$

To obtain the correct angular momentum quantization rule, relativistic invariance as a dynamic selection principle must be invoked. This requires a more complete classical mechanical description, which has been given by Bopp [22]. We introduce the four-vector of the velocity

$$u_\alpha = \mathrm{d}x_\alpha/\mathrm{d}s \equiv \dot{x}_\alpha, \quad \mathrm{d}s = \sqrt{1 - \beta^2}\, \mathrm{d}t , \tag{9.8}$$

where $\beta = v/c$, $x_\alpha = (x_1, x_2, x_3, ict)$ and where

$$Q = u_a^2 = - c^2 . \tag{9.9}$$

By differentiation we have

$$u_\alpha \dot{u}_\alpha = 0, \; u_\alpha \ddot{u}_\alpha + \dot{u}_\alpha^2 = 0, \; u_\alpha \dddot{u}_\alpha + 3 \dot{u}_\alpha \ddot{u}_\alpha = 0, \dots \tag{9.10}$$

For the following it is convenient to use units where $c = 1$. Following Bopp, we use instead of (9.1) the variational principle

$$\delta \int \Lambda(x_\alpha, u, \dot{u}_\alpha)\, \mathrm{d}s = 0 \tag{9.11}$$

with (9.9) as subsidiary condition. With λ a Lagrange multiplicator, the Euler-Lagrange equations are

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\partial(\Lambda + \lambda Q)}{\partial u_\alpha} - \frac{\mathrm{d}}{\mathrm{d}s}\frac{\partial \Lambda}{\partial \dot{u}_\alpha}\right) - \frac{\partial \Lambda}{\partial x_\alpha} = 0 . \tag{9.12}$$

As the Lagrange function we choose

$$\Lambda = - k_0 + \tfrac{1}{2} k_1 \dot{u}_\alpha^2 - e \Phi_\alpha u_\alpha , \tag{9.13}$$

where k_0 and k_1 are constants and $\Phi_\alpha = \{A_x, A_y, A_z, i\varphi\}$. A and φ are a vector and scalar potential, as they were derived from analyzing the vortex waves. Finally, e is a coupling constant. By inserting (9.13) into (9.12) one has

$$\frac{\mathrm{d}}{\mathrm{d}s}(2\lambda u_\alpha + k_1 \ddot{u}_\alpha) = - e f_{\alpha\beta} u_\beta , \tag{9.14}$$

where

$$f_{\alpha\beta} = \partial\Phi_\beta/\partial x_\alpha - \partial\Phi_\alpha/\partial x_\beta . \tag{9.15}$$

Because $f_{\alpha\beta} u_\alpha u_\beta = 0$ we find

$$u_\alpha \frac{\mathrm{d}}{\mathrm{d}s}(2\lambda u_\alpha + k_1 \ddot{u}_\alpha) = 2\dot{\lambda} u_\alpha^2 + 2\lambda u_\alpha \dot{u}_\alpha + k_1 \ddot{u}_\alpha u_\alpha = 0 \tag{9.16}$$

or because of (9.10)

$$-2\dot{\lambda} = 3k_1 u_\alpha \ddot{u}_\alpha = -2\dot{\lambda} - \frac{3}{2} k_1 \frac{\mathrm{d}}{\mathrm{d}s}(\dot{u}_\alpha^2) = 0 \tag{9.17}$$

hence (summation over v):

$$2\lambda = k_0 - \frac{3}{2} k_1 \dot{u}_v^2 , \tag{9.18}$$

where k_0 appears here as a constant of integration. Insertion of (9.18) into (9.14) leads to

$$\frac{\mathrm{d}}{\mathrm{d}s}\left[\left(k_0 - \frac{3}{2} k_1 \dot{u}_v^2\right) u_\alpha + k_1 \ddot{u}_\alpha\right] = -e f_{\alpha\beta} u_\beta . \tag{9.19}$$

We claim that (9.19) is the classical equation of motion for a pole-dipole particle coupled by the charge e to a four-vector field. This can be most easily demonstrated for the field-free case $f_{\alpha\beta} = 0$. It leads to

$$\frac{\mathrm{d}P_\alpha}{\mathrm{d}s} = 0, \quad P_\alpha = \left(k_0 - \frac{3}{2} k_1 \dot{u}_v^2\right) u_\alpha + k_1 \ddot{u}_\alpha , \tag{9.20}$$

where P_α are the components of the momentum-energy four-vector. For $k_1 = 0$ one has $p_\alpha = k_0 u_\alpha$, which by putting $k_0 = m$ is the four-momentum of a spinless particle with rest mass m.

The mass-dipole moment is given by

$$p_\alpha = k_1 \dot{u}_\alpha , \tag{9.21}$$

as can be seen from the conservation of angular momentum

$$\frac{\mathrm{d}}{\mathrm{d}s} J_{\alpha\beta} = 0 , \tag{9.22}$$

where

$$J_{\alpha\beta} = [\boldsymbol{x}, \boldsymbol{P}]_{\alpha\beta} + [\boldsymbol{p}, \boldsymbol{u}]_{\alpha\beta} \tag{9.23}$$

and where $[\boldsymbol{x}, \boldsymbol{P}]_{\alpha\beta} = x_\alpha P_\beta - x_\beta P_\alpha$. For a particle at rest ($P_k = 0$, $k = 1, 2, 3$) one has

$$J_{kl} = [\boldsymbol{p}, \boldsymbol{u}]_{kl} = p_k u_l - p_l u_k , \quad k, l = 1, 2, 3 , \tag{9.24}$$

which is just the spin angular momentum.

The energy of a pole-dipole particle at rest, and for which $u_4 = \gamma$, is determined by the fourth component

$$P_4 = i m = i(k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2)\gamma . \tag{9.25}$$

Alternatively, the energy can also be obtained from

$$P_\alpha u_\alpha = -\gamma m = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2) u_\alpha^2 + k_1 \ddot{u}_\alpha u_\alpha = -(k_0 - \tfrac{1}{2} k_1 \dot{u}_v^2) . \tag{9.26}$$

The mass m must, therefore, obey the double equation:

$$m = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2)\gamma = (k_0 - \tfrac{1}{2} k_1 \dot{u}_v^2)\gamma^{-1} . \tag{9.27}$$

The limit $v \to c$, that is $\gamma \to \infty$, corresponds to the Dirac equation. To keep m finite, one must, therefore, have in this limit

$$\lim_{\gamma\to\infty} (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2) \to 0, \qquad \lim_{\gamma\to\infty} (k_0 - \tfrac{1}{2} k_1 \dot{u}_v^2) \to \infty . \tag{9.28}$$

Therefore, $k_0 \to (\tfrac{3}{2}) k_1 \dot{u}_v^2$ and $k_0 \to \infty$. This means that the term in (9.20) multiplied with k_0 and which would be left if the mass dipole goes to zero, would become infinite. This infinite self-energy is similar to the infinite self-energy of a point charge in Maxwell's theory.

From (9.20) with $P_k = 0$, $k = 1, 2, 3$ follows for a circular orbit of radius r_c

$$p = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2) r_c \tag{9.29}$$

or because of (9.25) that

$$p = m r_c / \gamma \tag{9.30}$$

which is the same as (9.6). With $\boldsymbol{u} = \gamma \boldsymbol{v}$, we obtain for the spin angular momentum

$$J_z = -p u = -m v r_c \simeq -m c r_c \tag{9.31}$$

and which is the same as (9.7). The close relationship between the simple pole-dipole model and the Lagrange function (9.13) is, therefore, established, and the latter for this reason can serve for the quantized version.

With $L = \Lambda \, \mathrm{d}s/\mathrm{d}t$, Hamilton's variational principle (9.1) leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_k} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \ddot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0 . \tag{9.32}$$

Introducing the momentum and force components

$$P_k = \frac{\partial L}{\partial \dot{q}_k} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \ddot{q}_k} , \quad X_k = \frac{\partial L}{\partial q_k} , \tag{9.33}$$

and

$$R_k = \partial L / \partial \ddot{q}_k \tag{9.34}$$

the Hamilton function is obtained by the Legendre transformation:

$$H = -L + \sum_k \dot{q}_k P_k + \sum_k \ddot{q}_k R_k \,. \tag{9.35}$$

One, therefore, obtains for the canonical equations

$$\dot{P}_k = -\frac{\partial H}{\partial q_k}, \quad \dot{q}_k = \frac{\partial H}{\partial P_k}, \quad \dot{R}_k = -\frac{\partial H}{\partial \dot{q}_k}, \quad \ddot{q}_k = \frac{\partial H}{\partial R_k}. \tag{9.36}$$

The first two of these equations are the usual Hamilton equations of motion, and they describe the macromotion of the pole-dipole particle. The remaining two equations then describe the internal "Zitterbewegung" micro-motion. Applied to the Lagrange function $L = \Lambda\, \mathrm{d}s/\mathrm{d}t$, Bopp [22] finds for the Hamilton function (9.35)

$$H = -e\varphi + (\boldsymbol{v}, \boldsymbol{P} + e\boldsymbol{A}) + k_0\sqrt{1-v^2} - (1/2k_1)\sqrt{1-v^2}\,(\boldsymbol{\vartheta}^2 - (\boldsymbol{\vartheta}\,\boldsymbol{v})^2)\,, \tag{9.37}$$

where

$$\boldsymbol{P} = \frac{\partial L}{\partial \boldsymbol{v}} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{\boldsymbol{v}}}, \quad \boldsymbol{\vartheta} = \frac{\partial L}{\partial \dot{\boldsymbol{v}}} \tag{9.38}$$

and with the canonical equations

$$\dot{\boldsymbol{P}} = -\frac{\partial H}{\partial \boldsymbol{r}}, \quad \dot{\boldsymbol{r}} = \frac{\partial H}{\partial \boldsymbol{P}}, \quad \dot{\boldsymbol{\vartheta}} = -\frac{\partial H}{\partial \boldsymbol{v}}, \quad \dot{\boldsymbol{v}} = \frac{\partial H}{\partial \boldsymbol{\vartheta}}. \tag{9.39}$$

$\boldsymbol{P}$ and $\boldsymbol{r}$ are here the macrovariables and $\boldsymbol{\vartheta}$ and $\boldsymbol{v}$ the microvariables. With these variables, the angular momentum conservation law assumes the form

$$\boldsymbol{r} \times \boldsymbol{P} + \boldsymbol{v} \times \boldsymbol{\vartheta} = \text{const.} \tag{9.40}$$

The first term is there the external angular momentum of the macromotion and the second term the internal spin-type angular momentum of the micro-motion.

The Hamilton function (9.37) is of the same form as the Dirac Hamiltonian if one puts

$$\boldsymbol{P} \to \frac{\hbar}{i}\frac{\partial}{\partial \boldsymbol{r}}, \quad \boldsymbol{v} \to \boldsymbol{\alpha}, \quad \sqrt{1-v^2} \to \alpha_4\,, \tag{9.41}$$

where $\alpha = (\boldsymbol{\alpha}, \alpha_4)$ are the Dirac matrices. In the absence of an external field, $\varphi = \boldsymbol{A} = 0$, one obtains the Dirac equation

$$\frac{\hbar}{i}\frac{\partial \psi}{\partial t} + H\psi = 0\,, \tag{9.42}$$

where

$$H = \alpha_1 P_1 + a_2 P_2 + \alpha_3 P_3 + \alpha_4 m \tag{9.43}$$

with

$$\alpha_\mu^2 = 1; \quad \alpha_{\mu\nu} + \alpha_{\nu\mu} = 0, \ \mu \neq \nu \tag{9.44}$$

and where the mass is given by

$$m = k_0 - (1/2k_1)(\boldsymbol{\vartheta}^2 - (\boldsymbol{\vartheta}\cdot\boldsymbol{v})^2). \tag{9.45}$$

The introduction of negative masses and invoking Lorentz invariance as a dynamic selection principle has finally led to Dirac's equation. It, therefore, sets the spin quantization rule by putting

$$-J_z = \tfrac{1}{2}\hbar \tag{9.46}$$

with J_z given by (9.31). As a result we have

$$r_c = \hbar/2mc\,. \tag{9.47}$$

The mass term given by (9.45) has a rather complicated nonlinear composition. As it has been shown by Bopp, it leads to a mass spectrum if one solves the wave equation for the internal micro-motion. A hint for the origin of the mass is given by combining (9.7) with (9.5), which leads to the relation

$$m = m_0/\gamma\,. \tag{9.48}$$

The Dirac equation corresponds to the limit $\gamma \to \infty$. This can best be seen by the relation $\boldsymbol{v} = \boldsymbol{\alpha} c$, and which results in $v^2 = c^2$, for the "Zitterbewegung"-velocity. In the limit $\gamma \to \infty$, $m_0 \to \infty$ to keep m finite, but if $\gamma = \infty$ is not reached, then the mass m_0 can be very large. But because $m_0 = m^+ - |m^-|$, with $m_0 \ll m^+$, the positive and negative masses producing the mass of the Dirac spinor can be even larger. This unique property is only possible in the presence of negative masses, and it avoids the energy-mismatch problem due to the uncertainty principle for all preon models working with positive masses only.

10. Spinor Masses

We now propose that spinors are excitons of the substratum, made up from the positive and negative masses of the vortex resonance (vortons). As in solid state physics, an exciton is here too a quasiparticle which can move like a real particle by resonance excitation through the substratum. According to (8.6) the vortex resonance has the mass

$$m_v^{\mp} \simeq \pm m_p (r_p/R)^2\,. \tag{10.1}$$

As we had demonstrated in (5.9), the fluctuating Planckions attached to the vortex rings become the source of a scalar field which is equal to Newtonian gravitation between the Planck masses.

Following Hönl and Papapetrou [21], we, therefore, may consider a configuration of two masses $m_v^{\pm}$ of equal magnitude but opposite sign, with the mass pole m_0 coming from their gravitational interaction energy, which for a mass dipole is positive. This assumption is consistent with our model where the fundamental interaction is determined by the gravitational constant. This gravitational energy is

$$m_0 c^2 = G\,|m_v^{\pm}|^2/r\,. \tag{10.2}$$

Because of (9.48) we then have for the spinor mass

$$m c^2 = G\,|m_v^{\pm}|^2/\gamma r\,. \tag{10.3}$$

With $p = |m_v^{\pm}|\,r$, (9.6) and (9.47), one obtains

$$2\gamma\,|m_v^{\pm}|\,r c = \hbar\,, \tag{10.4}$$

and eliminating r from (10.3) and (10.4) one finds

$$m = 2G\,|m_v^{\pm}|^3/\hbar c = 2\,|m_v^{\pm}|^3/m_p^2\,. \tag{10.5}$$

Finally, eliminating $|m_v^{\pm}|$ from (10.1) and (10.5), we obtain the important result that

$$m/m_p = 2(r_p/R)^6\,. \tag{10.6}$$

We note that $m < m_p$, and that the mass is not quantized in units of m_p, as one naively might expect. The reason, of course, is that the mass m is the mass of the gravitational field set up in between the positive and negative masses $m_v^{\pm}$. In our model fields are reduced to mechanical properties of the ether and masses much smaller than m_p can always result from a slight imbalance in the kinetic fluid energy of the positive and negative mass ether.

To obtain $m/m_p \simeq 2\times 10^{-22}$, with m set equal the value for the lowest quark mass, would require to make $R/r_p \simeq 4600$, a value about 10 times larger than our estimate $R/r_p \simeq 500$. For the vorton mass we would have $|m_v^{\pm}| \simeq 10^{-12}$ g, with $|m_v^{\pm}|c^2 \simeq 6\times 10^{11}$ GeV. From (10.4), assuming that $r \sim R$ it then would follow that $\gamma \approx R/2r_p \sim 10^3$. Because of this large value for γ, these spinor-type excitons move through the substratum with a Zitterbewegung close to the velocity of light, but whereby the center of mass velocity can assume any value less than c.

Equation (10.6) permits us to express the gravitational constant in terms of the spinor mass and the ratio r_p/R. We find

$$G = (2/m)^2\,\hbar c\,(r_p/R)^{12}\,. \tag{10.7}$$

The masses of the different particle families could be explained as excited states of the pole-dipole configuration, involving radial pulsations. Bopp [22] obtained for the mass ratio of the first excited state to the ground state a value of the order 200. This value is in good agreement with the μ-meson electron mass ratio. The cause for this large mass ratio is the nonlinearity of the mass term (9.45). That these excited states are radial pulsations would explain why decays to a lower family must be rare.

The occurrence of a "desert" in particle physics is in our model explained by (10.6). It results from the gravitational binding of two very large masses of opposite sign, to obtain the much smaller masses of elementary particles.

Our computation of the typical elementary particle mass in terms of the Planck mass is incomplete for two reasons: First, if an elementary particle is electrically (or magnetically) charged, the mass resulting from these fields must be accounted for. Second, for very large gravitational fields, nonlinear effects diminish the overall gravitational field energy, resulting in a smaller mass. The electric field energy can certainly not be neglected for charged leptons, and the inclusion of the nonlinear gravitational effects might be responsible for the small lepton masses like that of the neutrino. If spinors are explained as bound states of positive and negative masses, then even the neutrino must have a finite rest mass which nevertheless can be very small. Because of the $|m_v^{\pm}|^3$ dependence for m, a small relative change in the vorton mass can lead to a large change in m. The inclusion of charges and their fields would imply a fine structure of the vorton and which may be responsible for the a large difference between the lepton and quark masses. These mass differences are still very small if compared with the Planck mass.

11. Discussion

One immediate question coming up is how this proposed theory of elementary particles can be related to the standard model. In trying to answer this question, we first note that our theory correctly recovers its two fundamental symmetries, the gauge and Lorentz invariance. Because these symmetries are in our theory dynamic, composite structures satisfying these symmetries are selected as stable bound states. The occurrence of specific other symmetries, for example those describing quarks would have to be explained by the

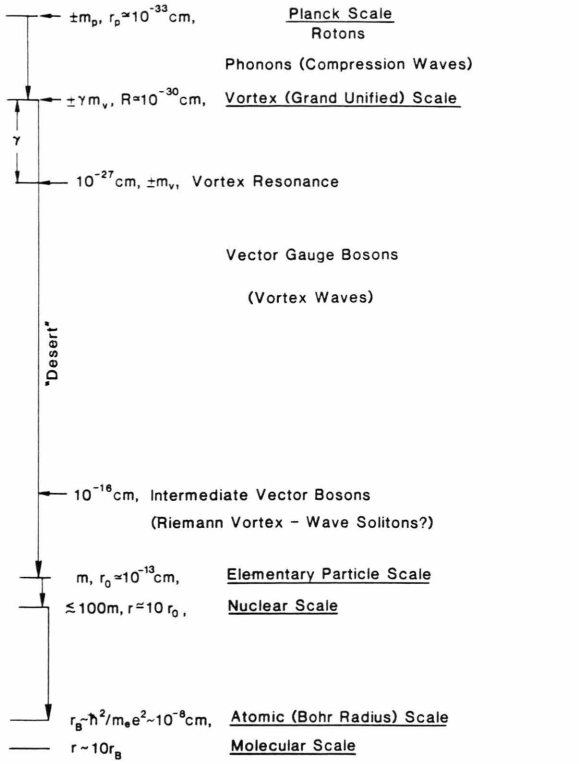

Fig. 5. The different scales and particles associated with these scales.

fine structure of the vortons and their interactions, very much like the multitude of crystal structures which result from the details of the atomic interactions. To obtain the many details of the standard model from the more fundamental and simpler structure at the vortex scale, can therefore hardly be less difficult than to get the many crystal structures from the dynamics of the much simpler spherical symmetric Coulomb forces determining the interaction between atoms.

From the perspective that all elementary particles are bound states of the substratum, three principal hierarchies emerge:

I. The Planck mass scale
II. The vortex grand unified scale
III. The elementary particle scale

In addition, there are at least three more scales:

IV. The nuclear scale
V. The atomic scale
VI. The molecular scale

In Fig. 5 we have displayed the different scales.

The zero rest mass gauge bosons, like the photon, are quantized vortex waves of the substratum. They play a similar role as the phonons in a solid, and for this reason are not considered elementary particles. Because all the different vector gauge bosons, being different quantized manifestations of the same vortex wave, are unified at the vortex scale R, this scale plays here the same role as the grand unification scale in the standard model. With the ratio $R/r_p \sim 10^3$, it now becomes understandable why the grand unification length is a few thousand times larger than the Planck length.

According to our model all interactions should be communicated by vector gauge bosons as quantized vortex waves. This conclusion seems to contradict the belief that gravity is communicated by quantized spin 2 tensor waves. However, in the presence of a fluidlike positive-negative mass substratum, it is possible to formulate a vector theory of gravity which reproduces all the experimentally tested linear and nonlinear effects of Einstein's tensor theory of gravity [24]. Nevertheless, by going to the equations for the fourpoint velocity correlations, even transverse tensor waves can be obtained from the substratum hydrodynamics, in the same way as transverse vector waves can be obtained from the two point velocity correlation tensor. According to Weinberg [25] a tensor field theory necessarily leads to Einsteins's theory in the low energy limit.

The quantum electrodynamic Lagrangian derived from our model is valid as well only in the low energy limit. This can be seen as follows. From the dynamic interpretation of special relativity and its application as a dynamic selection principle, the general Lagrangian, composed of the four-vector from the vortex wave field and the Dirac spinor derived from the pole-dipole configuration of the positive and negative mass vortex resonance energy, should have the form of an infinite series [26]:

$$
\begin{aligned}
L = & -\bar{\psi}\left(\gamma^{\mu} \frac{\partial}{\partial x^{\mu}} + m\right)\psi - \frac{1}{4}\left(\frac{\partial A_{\nu}}{\partial x^{\mu}} - \frac{\partial A_{\mu}}{\partial x^{\nu}}\right)^2 \\
& + i e_0 A_{\mu} \bar{\psi} \gamma^{\mu} \psi \qquad (11.1) \\
& + e_1 \left(\frac{\partial A_{\nu}}{\partial x^{\mu}} - \frac{\partial A_{\mu}}{\partial x^{\nu}}\right) \bar{\psi} \sigma^{\mu\nu} \psi + e_2 \bar{\psi} \psi \bar{\psi} \psi + \dots .
\end{aligned}
$$

In this expansion e_0 is a dimensionless coupling constant, but the coupling constant e_1 has the dimension $[\text{mass}]^{-1}$ and e_2 the dimension $[\text{mass}]^{-2}$. Coupling constants for higher terms not written down, would have even larger negative dimensions in mass. The Maxwell-type equations for the vector gauge field and the Dirac equation were derived under the assumption that the wave length is large compared to R. This is obvious for the vortex waves. For the pole-dipole configuration it expresses itself in the restriction of Λ to terms not higher than $\dot{u}_\alpha^2$. In general Λ could have terms proportional to $\ddot{u}_\alpha^2$, $\dddot{u}_\alpha^2 \ldots$. It is therefore suggested that $e_1 \sim M_G^{-1}$ and $e_2 \sim M_G^{-2}$, where $M_G = \hbar/Rc$ is the mass of the grand unified vortex scale, and which means that in comparison with the first three terms, all the remaining terms in the Lagrangian (11.1) are extremely small. Therefore, as long as the energies mc^2 are small in comparison to $M_G c^2 \sim 10^{16}$ GeV, the Lagrangian of quantum electrodynamics is recovered as an extremely good approximation.

One prediction of our theory which appears to be in remarkable good agreement with the experiment is the relation in between G, m and the ratio r_p/R, expressed by (10.7). According to our theory one would have to set $R/r_p = m_p/M_G$, and (10.7) predicts a value $m_p/M_G \simeq 4600$, which within a factor 2 is equal the ratio of the Planck to the grand unified scale obtained from the extrapolation to high energies of the running coupling constants determined experimentally at low energies.

Quantum electrodynamics is extremely well tested experimentally, but the same cannot be said about quantum chromodynamics*. In fact, several experiments clearly contradict the predictions of quantum chromodynamics [27, 28]. To explain the dynamic behavior of quarks, new charges, called "color", have been introduced. In addition, magnetic monopole charges might also exist. However, it is quite possible that all these charges are different manifestiations of the same fundamental Planckion mass charge for the following reason: The vortex core to which the Planckion charges are attached holds roughly R/r_p Planckion charges. At a distance large compared to the vortex ring radius, the observed charge may be much smaller due to vacuum polarization, caused by opposite mass charges shielding the charge of one vortex. Now, if two opposite charges g, bound in a dipole, and for which

$$g^2 > G m_p^2 = \hbar c \tag{11.2}$$

are separated, they each would pull out of the vacuum, resp. the substratum, a monopole g of opposite sign, resulting in the formation of two dipoles but of no monopoles. A Planckion charge for which $g > \sqrt{\hbar c}$ would, therefore, behave very much like a magnetic monopole, for which according to Dirac's quantization rule

$$e g = \hbar c \tag{11.3}$$

where $e = \sqrt{\hbar c/137}$ is the electric charge. With this assignment, it is then possible, as it was shown by Schwinger [29], to explain color as a kind of magnetic charge, and the quarks as dually charged particles. The gravitational interaction would then play the role of a geometric average

$$\sqrt{G}\, m_p = \sqrt{e g}\,. \tag{11.4}$$

Below the vortex scale the interactions separate into a strong (g), intermediate ($\sqrt{G}\, m_p$) and weak (e) part*. This behavior can be qualitatively understood by assigning for the strong and weak interaction two different vortex waves communicated by vortices holding a large and a small number of Planck charges.

It is of particular interest that our model provides for an intermediate mass scale $|m_v^{\pm}|\, c^2 \simeq 10^{12}$ GeV, because the existence of such a scale has been demanded to explain the small neutrino masses ("seesaw mechanism" by Gell-Mann, Ramond, and Slansky).

In the standard model, it is the Higgs field which is believed to give mass to all elementary particles, but in particular to the intermediate vector bosons. In the theory proposed here, the intermediate vector boson would acquire mass through soliton solutions of the vortex wave equation for large amplitudes, and fermions would get their mass through the field energy in positive-negative mass composite structures. However, in either case the mechanism by which mass is acquired is through the nonlinear ψ^3-term in the fundamental equation, which has the same form as the corresponding selfcoupling φ^3-term in the Higgs field equation. In the standard model the φ^3-term in the Higgs field equation leads to a huge cosmological con-

* The occurrence of fractional charges in quantum chromodynamics, quantized in units $e/3$, may have the same cause as the "1/3-effect" observed in the quantized Hall effect, which results there from the formation of quasiparticles with an effective charge $e/3$ out of a two-dimensional electron fluid [35]. It happens that a two-dimensional symmetry is just realized in three-quark configurations, with the three quarks always lying in a plane.

* We give the interactions here different names than those used in the standard model.

stant, which would curl up the universe to football size dimensions, something which is obviously not the case. No such problem exists here. In the standard model the working of the Higgs mechanism is explained by its adjustment to satisfy Lorentz invariance. In our theory this is required on dynamic grounds. It is worthwhile to mention that there appears to be a direct relation of the vortex structure and the Higgs field. As it was shown by Nielsen and Olesen [30], a relativistic Higgs field coupled to a Maxwell type vector gauge field leads to vortex-like solutions. Therefore, if the vortices in the superfluid ether lead through their interaction to transverse waves satisfying Maxwell's equations, this situation could as well be described by vector gauge bosons coupled to a Higgs field.

A comparison of our theory with Heisenberg's nonlinear spinor theory has been already made. Of interest is also a comparison with v. Weizsäcker's "ur" theory [31]. It demands the $U(2)$ group as the fundamental symmetry**. Since $U(2) = U(1) \times SU(2)$ this demand is fulfilled by our fundamental equation.

Still more revealing is a comparison of our theory with superstring theories. One problem for these theories is that they have only one scale, and which is the Planck scale. Because relativity requires to set the string diameter equal to zero, the string radius is set equal the Planck length. One should, therefore, expect that the grand unification length is approximately equal to the Planck length [32]. From the measured data, however, the grand unification length turns out to be 10^3 to 10^4 times larger than the Planck length [33]***. By comparison, the vortices in the superfluid ether have a ring size much better in agreement with the grand unification scale. The need to make string theories supersymmetric (the reason why they are called superstring theories) to enlarge their symmetry to account for fermions, is unnecessary in our theory, where Dirac spinors arise from the presence of negative masses in the ether. When confronted with reality, superstring theories have a number of serious problems. To make a string theory unique, requires that it be formulated in a higher (for example, 10) dimensional space, in gross disagreement with the physical reality of a four-dimensional space-time. To account for this discrepancy the superfluous dimensions must be compactified down to the Planck scale. But this destroys the uniqueness of the theory, with several thousand different ways the compactification can be accomplished. More recently string theories in four dimensions have been proposed, but there the problem is that a very large number of string theories are then possible, and as a result one is really not better off. Another problem for string theories is that relativistic invariance requires the string to have a zero diameter. Because the string shall be held together by a force, this implies an infinite stress in the string. From a physical point of view, this is very implausible, where anything which turns out to be infinite must be viewed with suspicion as a mathematical artifact. Topologically, strings and vortex rings are quite similar, and we note that in heterotic superstring theories the charges are spread out over the closed string, very similar to our theory, where the Planckion charges are distributed along the vortex core. It is for this reason that string theories may turn out to be an approximation for the substratum theory with vortex rings.

One of the most puzzling aspects of high energy physics is the phenomenon of parity violation in weak interaction. Shortly after its discovery it was believed that its cause is a vanishing rest mass of the neutrino. A zero rest mass neutrino sustains its helicity and nature could have selected (for reasons unknown) neutrinos of just one helicity. This conjecture has been shown wrong. From high energy physics experiments we now know that the cause of parity violation lies in the intermediate vector bosons, because these bosons interact more strongly, not only with left handed neutrinos, but also with left handed electrons. The latter have certainly a finite rest mass. A vanishing rest mass of the neutrino would also contradict the pole-dipole particle model, with the radius of gyration given by (9.47) becoming infinite. In our model parity violation might be caused by spontaneous symmetry breaking in the superfluid substratum through the formation of vortex filaments along a preferred direction in space. These vortex filaments would resemble the current vortices in type II superconductors, and would be superimposed on the ring vortices. If these vortex filaments, with the vorticity $\boldsymbol{\omega}$ related to the substratum

** Von Weizsäcker has tried to derive Lorentz invariance as a kinematic symmetry from the $U(2)$ symmetry, but the proof presented is not very convincing.

*** In a recent popular article [34] about superstring theories written by one of its inventors, M. B. Green, the value for the grand unification length in a diagram is there shown only 100 times larger than the Planck length. Such a much smaller length is, of course, better in line with the expectation of superstring theories, but hardly supported by the empirical evidence.

velocity $\boldsymbol{v}$ by

$$\omega = (1/2)\,\mathrm{curl}\,\boldsymbol{v} \tag{11.5}$$

transmit their vorticity to the intermediate vector bosons, parity would be violated. This highly speculative conjecture would make it understandable why parity violation can only occur in conjunction with massive particles, because only they would have a Compton wavelength small enough to feel the microscale vorticity of the substratum. The direction and strength of vorticity may change with a direction in space, and be different in other parts of the universe.

Likewise, CP violation could perhaps be explained in a slight asymmetry of the substratum vorticity, by a small difference between the vorticity of the positive and negative masses of the substratum.

As we had remarked, the vortex lattice of the substratum may be generated by spontaneous symmetry breaking through the interaction of the positive and negative masses, resulting in rapid circular motion caused by the same mechanism leading to the Zitterbewegung of the pole-dipole configuration, that is by a Zitterbewegung of the ether. Because the core radius of the vortices is equal the Planck length, the spontaneous symmetry breaking must occur near the upper end of the energy spectrum, where the rotons are located and which until now we have ignored. Since the rotons can be seen as vortices in "statu nascendi" they are likely to play an important role in the spontaneous symmetry breaking mechanism, finally establishing also their significance.

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